MATH 135 — QUIZ 6 — JAMES HOLLAND
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Question 1. Using implicit differentiation, find the tangent line to $y = (1-x)^{1-x}$ at the point $(0, 1)$.

Solution .:.

Taking the natural logarithm of both sides of the equation yields that $\ln(y) = \ln((1-x)^{1-x}) = (1-x)\ln(1-x)$. Implicit differentiation then yields that

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = -1\ln(1-x) - \frac{1-x}{1-x} = -\ln(1-x) - 1.$$

Therefore $\frac{\mathrm{d}y}{\mathrm{d}x} = -y\ln(1-x) - y$. At the point $(0, 1)$, it follows that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\ln(1) - 1 = 0 - 1 = -1$. Hence the slope of the tangent line is -1. We are given the point $(0, 1)$, so the tangent line is given by $y = -x + 1$.

Question 2. For x in $[-2, 7]$, calculate the minimum and maximum value of y where $y^3 = x + 1$.

Solution .:.

$y = (x+1)^{1/3}$, which means $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}(x+1)^{-2/3}$. This is never 0, but it is undefined when $x = -1$. Hence we will investigate y when x is -2, -1, and 7:

$$(-2+1)^{1/3} = -1 \qquad (-1+1)^{1/3} = 0 \qquad (7+1)^{1/3} = 8^{1/3} = 2.$$

So -1 is the minimum value of y, and 2 is the maximum value of y.

Alternatively, by implicit differentiation, $3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 1$. Hence $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3y^2}$, which is never 0, and is undefined iff $y = 0$, i.e. $0^3 = x + 1$ meaning $x = -1$. So $x = -1$ is the only critical point, and so we investigate $x = -2$, $x = -1$, and $x = 7$:

- $y^3 = -2 + 1 = -1$ implies $y = -1$;
- $y^3 = -1 + 1 = 0$ implies $y = 0$;
- $y^3 = 7 + 1 = 8$ implies $y = 2$.

So $y = -1$ is the minimum value, and $y = 2$ is the maximum value.